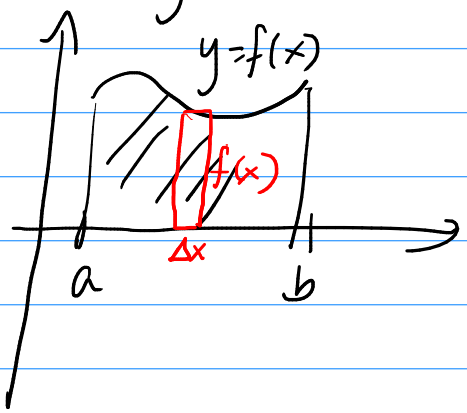
Lesson 12 Area between Curves, part I

I. Area Review

II Area between $y=f(x)$ and $y=g(x)$

I. Area Review

Area underneath $y=f(x)$ from $x=a$ to $x=b$



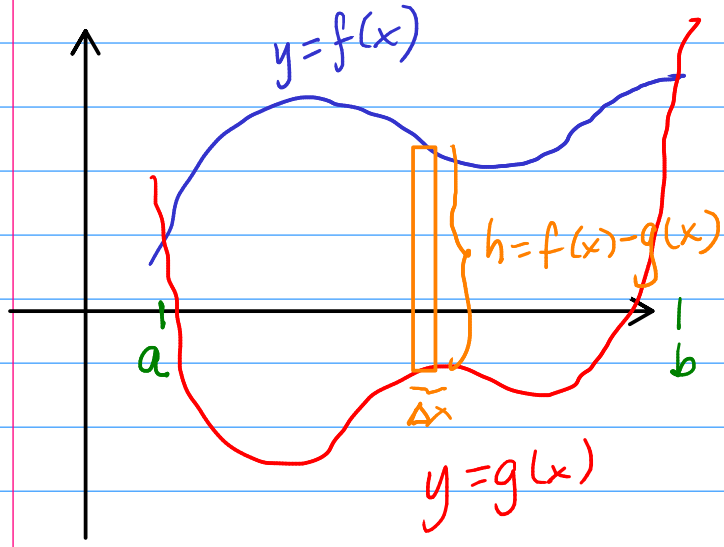
$$A=\int_a^b f(x)\,dx$$

height width

To find area (2-D), you integrate height. (1-D)

Same problem Find the area between $y=f(x)$ and $y=0$ (x-axis) from $x=a$ to $x=b$.

II. Area between curves $y=f(x)$ and $y=g(x)$

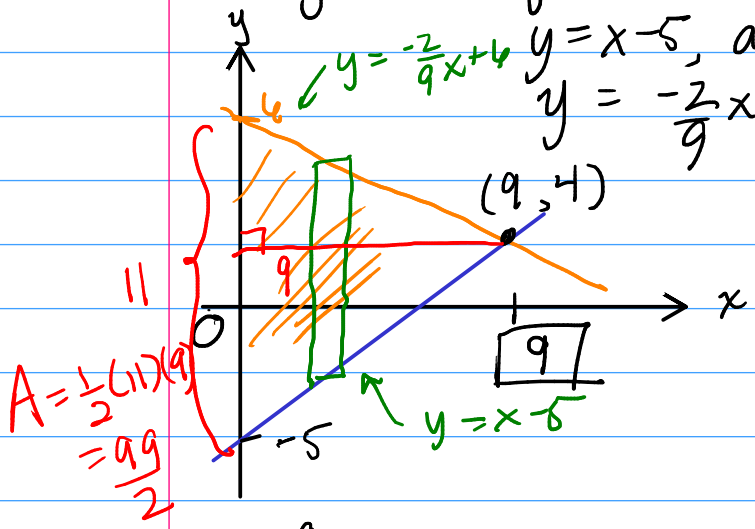


① Need to find a & b

② height = big fctn − small fctn
= top − bottom

for y = fctns of x

Ex Find the area of the triangular region bounded by the y-axis, ($x=0$) $y=x-5$, and $y=-\frac{2}{9}x+6$



$A=\frac{1}{2}(11)(9)=\frac{99}{2}$

Find pts of intersection

$$x-5=-\frac{2}{9}x+6$$

$$9x-45=-2x+54$$

$$11x=99$$

$$x=9$$

$$A=\int_0^9\left[\left(-\frac{2}{9}x+6\right)-(x-5)\right]dx=\int_0^9\left(-\frac{11}{9}x+11\right)dx$$

$$=-\frac{11}{9}\frac{x^2}{2}+11x\Big|_0^9=-\frac{11}{9}\cdot\frac{9^2}{2}+11(9)-(0+0)$$

$$=-\frac{99}{2}+99=\frac{99}{2}$$

EX Find the area between $y = x^2$ and $y = -x^2 + 18x$

intersection pts:

$x^2 = -x^2 + 18x$

$2x^2 = 18x$

$x^2 = 9x$

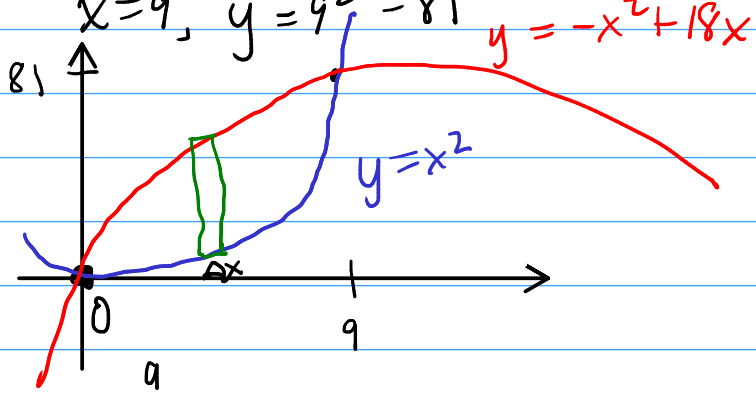
$x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0, \; x = 9$

$x = 0, \; y = 0^2 = 0$

$x = 9, \; y = 9^2 = 81$



$$A = \int_0^9 [-x^2 + 18x - x^2] \, dx = \int_0^9 [-2x^2 + 18x] \, dx$$

$$= \frac{-2x^3}{3} + \frac{18x^2}{2} \Big|_0^9$$

$$= \frac{-2x^3}{3} + 9x^2 \Big|_0^9$$

$$= \frac{-2 \cdot 9^3}{3} + 9(9)^2 - (0 + 0)$$

$$= -486 + 729 = 243 \text{ un}^2$$

Ex Area between $y = \frac{16}{x}$ $y = 12 - 2x$

intersection pts

$$\frac{16}{x} = 12 - 2x$$

$$16 = 12x - 2x^2$$

$$8 = 6x - x^2$$

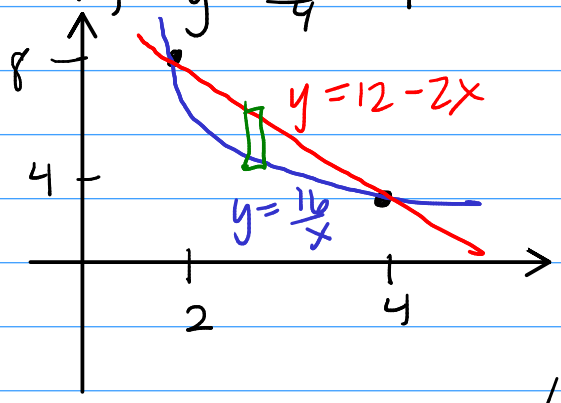
$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$x = 2$ $x = 4$

$x = 2$, $y = \frac{16}{2} = 8$

$x = 4$, $y = \frac{16}{4} = 4$



$$A = \int_2^4 \left[(12-2x) - \frac{16}{x}\right] dx$$

$$= 12x - \frac{2x^2}{2} - 16\ln(1x1)\Big|_2^4$$

$$= 12x - x^2 - 16\ln(1x1)\Big|_2^4$$

$$= 48 - 16 - 16\ln(4) - (24 - 4 - 16\ln(2))$$

$$= 12 - 16\ln(4) + 16\ln(2)$$